2024 Pacific Northwest Division 2 Solutions

The Judges

November 16, 2024

Big Integers

Problem

- Given two base-62 integers, compute which one is smaller.

Solution

- If the integers are different lengths, the one that is shorter is smaller.
- Otherwise, compare the characters by their base-62 values and find the first mismatch, then compare those directly.

Bus Assignment

Problem

- A bus travels along n stops where people get off the bus and get on the bus. Compute the minimum number of seats on the bus such that every person always has a seat on the bus.

Solution

- Keep track of the number of people on the bus at every point. The maximum number of people on the bus at some point is the answer.

Generalized Fizz Buzz

Problem

- Given three integers n, a, and b, count how many of the first n integers are divisible by both a and b, by a but not b, and by b but not a.

Solution

- n is small so we can check all of the integers from 1 to n directly.

Intuitive Elements

Problem

- Given two strings s and t, determine if every letter in t appears in s.

Solution

- Due to the low bounds, it's possible to naively loop over every character in t and loop over every character in s to check if it is present in both strings.
- For a faster solution, you can use a data structure like a set to track which characters exist in s.

Mouse Pursuit

Problem

- Given n events where each event involves either gaining or losing some amount of cheese and glory at some point in time, compute how much cheese and glory was gained or lost in the last k seconds.

Solution

- Loop over each event and see if it happened in the last k seconds, and maintain a running sum.
- Because the values are large, 64-bit integers are necessary.

Ruffians

Problem

- Given a 2×5 grid of single digit integers, determine if there exist a pair of identical integers in different rows and different columns.

Solution

- Because the grid is small, it is possible to brute force all pairs of locations in different rows and different columns.

Sleeping on the Train

Problem

- A train continuously travels from station 1 to station n and back. Given that the train is found between stations s and $s+1$ at different points in time, and someone boarded the train at station a and disembarked at station b, compute the minimum number of times the train turns around at either station 1 or station n.

Solution

- Carefully keep track of whether the train is moving towards increasing station numbers or decreasing station numbers.
- Every time there is an inconsistency found, increment the number of times the train must have turned around by 1 and change the direction of the train.

Champernowne Subsequence

Problem

- Compute the smallest positive integer k such that, if you write the first k positive integers in order, a given string of n digits appears as a subsequence.

Solution

- The answer for a string of length n is at most $10n$.
- If k is valid, then $k+1$ is valid, so we can binary search for the answer.
- Alternatively, we can just iteratively write the integers from 1 in order and keep track of how many of the first d digits we have written, and stop once we have written down all n digits.

Exact Change

Problem

- Given two integers a and b, compute the smallest cardinality multiset of integer powers of two such that every integer from a to b can be expressed as the sum of some subset of integers from the multiset.

Observation

- It is never optimal to have two coins of exactly the same denomination.

Solution

- If $a = b = 0$, the answer is zero.
- Consider the largest power of two less than or equal to b, 2^x.
- If $2^x \geq a$, then we can recursively solve this problem for the range $a - 2^x$ to $b - 2^x$.
- Otherwise, $2^x - 1$ must be representable, so the answer is $x + 1$ and the set $2^0, \ldots, 2^x$ suffices.

Homework Help

Problem

- There is a secret permutation of n integers. In a single query, you can request the number of inversions of a given subarray. Given the ability to make n queries, compute the length of the longest increasing subsequence of the array.

Solution

- Count the number of inversions for every prefix of the array. With this information, the exact array can be constructed exactly.
- We can then compute the length of the longest increasing subsequence in $\mathcal{O}(n^2)$. This can be done by maintaining the length of the longest increasing subsequence ending at a given index and computing all of these from left to right.

Rainbow Bowl Ranges

Problem

- There is a circle of n bowls and you have some balls in one of m colors. For each color, pick a contiguous range of bowls and put one ball in each bowl in the range. A bowl is a rainbow bowl if every color of ball is in the bowl. A rainbow bowl range is a maximal contiguous range of rainbow bowls. Compute the maximum number of rainbow bowl ranges that can be formed given the number of bowls of each colors.

Solution

- Since the colors must be contiguous, we can instead maintain the number of bowls that do not contain that color.
- If all colors appear n times, then the answer is obviously 1.
- Otherwise, an optimal arrangement involves using the least common color and then the k most common colors, such that the sum of the gaps, plus $k + 1$, is less than or equal to n.

Taking out the Trash

Problem

- Given n integers, we can remove either one integer from the list, or we can remove two integers as long as their sum is less than or equal to m. Compute the minimum number of operations needed to make the list empty.

Solution

- Consider the largest integer in the list. If we cannot remove it along with the smallest integer, we must remove it by itself. Otherwise, we can remove it along with the smallest integer.
- If we sort the list of integers, we can simulate this process in linear time.

Word Game

Problem

- Given n words, and a merge operation where two strings s and t can be combined if there exists a length-k suffix of s that is a length-k prefix of t, compute the lexicographically smallest word that can be generated by merging pairs of words until only one word is left.

Solution

- Due to the small value of n and the short length of strings, a recursive brute force solution that checks all possible ways of merging is fast enough.
- Note that if multiple values of k are valid, you must try all of them!